Independent Events

Example

1. Calculate the standard deviation of the binomial distribution with n trials and $p = \frac{1}{2}$.

2. I flip a coin and roll two die. What is the probability that I flip heads and roll snake eyes (two 1s)?

Problems

3. True   False   Let X be 1 the first card in a deck is an ace of spades and 0 otherwise. Let Y be 1 if the last card in the deck is a king of hearts and 0 otherwise. Then X, Y are independent random variables.

4. True   False   The expected values of the sum of any two random variables add only if the random variables are independent.

5. True   False   The variance of the sum of any two random variables add only if the random variables are independent.

6. True   False   I pick two random people from a crowd. Let X be the height of the first person and Y the height of the second. Then X, Y are independent.

Central Limit Theorem

Example

7. Show that the distribution of $\overline{X}$, the average of n i.i.d. random variables with mean μ and standard deviation σ has mean μ and standard deviation $\sigma/\sqrt{n}$.

8. Suppose that in the 2012 election, 55% of people preferred Obama over Romney. If I sample 100 random people (assume that they are independently chosen), what is the probability that a majority of them support Obama?

Problems

9. True   False   The example above proves the central limit theorem.

10. True   False   You can use the Central Limit Theorem to prove the Law of Large Numbers.

11. True   False   For a constant $c \geq 0$, we have that $SE(cX) = cSE(X)$.

12. True   False   Suppose I calculate that probability that in a sample of $10,000$ men, their average height is less than 66 inches is 99.9%. Then all but one or two men in a sample of $10,000$ men will have a height of less than 66 inches.

13. Suppose that the height of women is distributed with an average height of 63 inches and a standard deviation of 10 inches. Taking a sample of 100 women, what is the probability that the average of the heights of these 100 women is between 62 and 64 inches?

14. Suppose the weight of newborns is distributed with an average weight of 8 ounces and a standard deviation of 1 ounce. Today, there were 25 babies born at the Berkeley hospital. What is the probability that the average weight of these newborns is less than 7.5 ounces?

15. Suppose that the average lifespan of a human is 75 years with a standard deviation of 10 years. What is the probability that in a class of 25 students, they will on average live longer than 80 years?

16. The newest Berkeley quarterback throws an average of 0.9 TDs/game with a standard deviation of 1. What is the probability that he averages at least 1 TD/game next season (16 total games)?

17. Suppose that the average shopper spends 100 dollars during Black Friday, with a standard deviation of 50 dollars. What is the probability that a random sample of 25 shoppers will have spent more than $3000?

18. Suppose that on the most recent midterm, the average was 60 and the standard deviation 20. What is the probability that a class of 25 had an average score of at least 66?